

On the reversibility of the Meissner effect and the angular momentum puzzle

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It is generally believed that the laws of thermodynamics govern superconductivity as an equilibrium state of matter. Here we point out that within the conventional BCS-London description of the normal-superconductor transition in the presence of a magnetic field, the transition cannot be reversible, in contradiction with the thermodynamic description and with experiments. This indicates that the conventional theory of superconductivity is internally inconsistent. We argue that to describe a reversible transition it is necessary to assume that charge transfer occurs across the normal-superconductor phase boundary, as proposed in the theory of hole superconductivity. This provides also a solution to the angular momentum puzzle pointed out in previous work. Our explanation can only apply if the current carriers in the normal state are holes. An experimental test of these ideas is proposed.

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I. INTRODUCTION

The experimental discovery of the Meissner effect in 1933 [1] suggested that the transition between normal and superconducting states in the presence of a magnetic field is a reversible phase transformation between well-defined equilibrium states of matter to which the ordinary laws of equilibrium thermodynamics apply [2]. For example, the Rutgers relation [3] relating the specific heat jump between normal and superconducting phases at the critical temperature to the temperature derivative of the thermodynamic critical field follows from this description. In fact, the Rutgers relation had been found experimentally and interpreted theoretically using thermodynamics [4] before the discovery of the Meissner effect, in a sense anticipating it. Subsequent extensive experimental tests [5, 6] confirmed that in the ideal situation the normal-superconductor transition occurs without entropy production within experimental accuracy, i.e. is reversible, and this has been generally believed ever since.

In this paper we point out that within the conventional London-BCS theory of superconductivity [7] the transition between normal and superconducting states in the presence of a field *cannot* be reversible but instead is necessarily associated with entropy production. If so, this would render the usual thermodynamic description invalid, and indicate that the experiments consistent with reversibility were flawed [5, 6, 8]. However, we argue instead that the conventional London-BCS description of the transition is flawed, and that the transition *is* reversible and thermodynamics applies because of some physics that is absent in London-BCS theory but occurs in nature during the normal-superconductor transition: *charge transfer* in direction perpendicular to the normal-superconductor phase boundary.

In recent work we have argued that charge transfer in direction perpendicular to the normal-superconductor phase boundary is necessary to explain the dynamics of the Meissner effect [9, 10]. In this paper we show that this charge transfer is necessary to render the transition be-

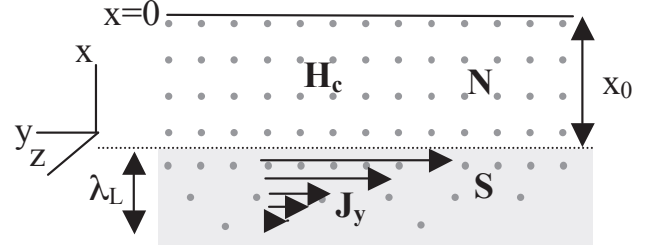


FIG. 1: Normal (N) and superconducting (S) phases in equilibrium. The N-S phase boundary is shown by the dotted line. The magnetic field H_c (indicated by grey dots) points out of the paper. The current flowing in the (-y) direction in the superconducting region within a London penetration depth (λ_L) of the phase boundary screens the magnetic field so that it is 0 beyond λ_L of the phase boundary. No current flows in the normal region.

tween normal and superconducting states reversible, and that the transition would be irreversible in the absence of this charge transfer, in contradiction with experiment. In addition, we show that this charge transfer resolves the angular momentum puzzle associated with the Meissner effect that we pointed out in previous work [11, 12]. Finally, we discuss an experimental test of these ideas.

II. PHASE EQUILIBRIUM

In a seminal paper [13], H. London analyzed the phase equilibrium between normal and superconducting states in the presence of a magnetic field. The situation is shown schematically in Fig. 1. Following the treatment and notation of ref. [9], in the superconducting phase ($x < x_0$) a current flows along the y direction parallel to the phase boundary located at $x = x_0$, given by

$$J_y(x) = -\frac{c}{4\pi\lambda_L} H_c e^{(x-x_0)/\lambda_L} \quad (1)$$

and correspondingly the magnetic field in this region $\vec{H}(x) = H(x)\hat{z}$ is given by

$$H(x) = H_c e^{(x-x_0)/\lambda_L} \quad (2)$$

so as to satisfy the London and Ampere equations

$$\vec{\nabla} \times \vec{J} = -\frac{c}{4\pi\lambda_L^2} \vec{H} \implies \frac{\partial J_y}{\partial x} = -\frac{c}{4\pi\lambda_L^2} H \quad (3a)$$

$$\vec{\nabla} \times \vec{H} = \frac{4\pi}{c} \vec{J} \implies \frac{\partial H}{\partial x} = -\frac{4\pi}{c} J_y. \quad (3b)$$

The system is at temperature $T < T_c$ and the thermodynamic critical field at that temperature is H_c . With the current given by $J_y = en_s v_s$, n_s the number of superconducting carriers of charge e per unit volume, and using the standard relation $1/\lambda_L^2 = 4\pi n_s e^2 / (mc^2)$ [7] the kinetic energy of the supercurrent per unit volume at $x = x_0$ is, from Eq. (1)

$$\epsilon_k = n_s \frac{1}{2} m v_s^2 = \frac{m J_y^2}{2e^2} = \frac{H_c^2}{8\pi}. \quad (4)$$

London [13] considered a virtual displacement of the phase boundary and derived an equilibrium condition for coexistence of the two phases

$$\Delta F = \frac{H_c^2}{8\pi} = n_s \frac{1}{2} m v_s^2 \quad (5)$$

where ΔF is the difference in free energy per unit volume between the normal and the superconducting phase. For the case where the phase boundary moves into the superconducting region the situation is shown schematically in Fig. 2. Eq. (5) says that the kinetic energy of the supercurrent at the phase boundary equals the superconducting condensation energy, and no energy is lost in irreversible processes. In this derivation London neglected the Joule heat Q that would necessarily be generated when the phase boundary is displaced and an electric field is induced according to Faraday's law, arguing that if the motion of the phase boundary is slow enough it can be neglected and hence the problem could be treated as a reversible phase transformation.

However, we argue that London's analysis is flawed because Joule heat is necessarily generated when the phase boundary moves, no matter how slowly the process is. Since Joule heat is only generated in the normal region, more energy will be dissipated when the phase boundary moves into the superconducting region and the normal region is enlarged than in the reverse process. Thus, the phase boundary will spontaneously move in the direction of enlarging the superconducting region and thermodynamic equilibrium will not result under the 'equilibrium condition' Eq. (5).

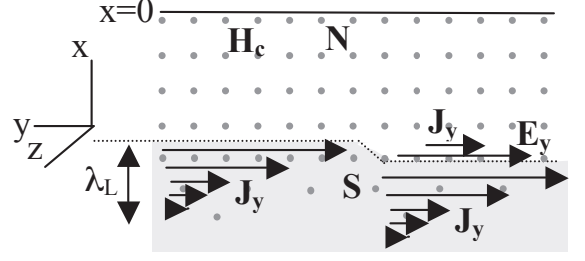


FIG. 2: The N-S phase boundary (dotted line) is displaced by a small amount in the negative \hat{x} direction, enlarging the normal region. This causes an increase in the magnetic flux in that region and an electric field E_y is necessarily generated, producing a current $J_y = \sigma E_y$ in the region becoming normal (σ = normal state conductivity).

III. REVERSIBILITY AND ENTROPY PRODUCTION

In thermodynamics a sharp distinction is made between quasistatic and reversible processes. Both processes have to happen slowly enough that the system can be considered to be in thermodynamic equilibrium at all times, so reversible processes are also quasistatic. However, a quasistatic process will be irreversible if it gives rise to an entropy increase. Instead in a reversible process entropy is not generated at any time.

We argue that the normal-superconductor transition in the presence of a magnetic field cannot be reversible within the conventional description. As the phase boundary is displaced and the magnetic flux changes, a Faraday field E_y will *necessarily* be generated which will give rise to a current J_y in the normal region, as shown schematically in Fig. 2. Thus, the normal-superconductor transition in the conventional understanding is necessarily irreversible, even if it can be argued that the irreversibility is very small if the transition occurs very slowly [13]. However, the speed at which the transition occurs is determined by the physical conditions in the system [9, 14]. In the next section we will estimate the degree of irreversibility that is expected in a typical situation and argue that it is incompatible with the numerous experimental reports that appeared to confirm reversibility of the normal-superconductor transition in a magnetic field.

IV. QUANTITATIVE ESTIMATE OF IRREVERSIBILITY

Consider the rectangular geometry shown in Fig. 3. The magnetic field points out of the paper and we assume for simplicity that it has magnitude H_c over the entire normal region of depth x_0 . Assume the phase boundary moves up at a uniform rate \dot{x}_0 , so that the magnetic field is expelled over a time

$$t_R = \frac{R}{\dot{x}_0} \quad (6)$$

where R is the initial length of the normal region in the x direction. The phase boundary position at time t is given by

$$x_0(t) = \dot{x}_0 t - R \quad (7)$$

According to Faraday's law, an electric field exists in the y direction in the normal region given by

$$E_y = \frac{H_c}{c} \dot{x}_0 \quad (8)$$

giving rise to a current $J_y = \sigma E_y$, with σ the normal state conductivity. The total Joule heat dissipated per unit time is

$$w(t) = J_y E_y [x_0(t)A] = \sigma E_y^2 [x_0(t)A] \quad (9)$$

over a volume $V(t) = x_0(t)A$, with A the cross-sectional area of the sample. Integrating over time we obtain for the total heat dissipated per unit volume

$$W = \frac{1}{RA} \int_0^{t_R} w(t) dt = \frac{\sigma E_y^2}{2} t_R = \frac{H_c^2}{8\pi} \frac{4\pi\sigma}{c^2} \frac{R^2}{t_R}. \quad (10)$$

Using a Drude form for σ

$$\sigma = \frac{ne^2\tau}{m_e} \quad (11)$$

with τ the Drude collision time, together with the usual relation for the London penetration depth [7]

$$\frac{1}{\lambda_L^2} = \frac{4\pi ne^2}{m_e c^2} \quad (12)$$

we can write Eq. (10) as

$$W = \frac{H_c^2}{8\pi} \left(\frac{R}{\lambda_L}\right)^2 \frac{\tau}{t_R} \quad (13)$$

For example, for $R = 1\text{cm}$, $\lambda_L = 500\text{\AA}$

$$W = \frac{H_c}{8\pi} (4 \times 10^{10}) \frac{\tau}{t_R} \quad (14)$$

so that for a typical collision time at low temperatures $\tau = 10^{-11}\text{s}$ if it took $t_R = 10\text{s}$ for the phase boundary to move 1cm the Joule heat dissipated is 4% of the condensation energy. In typical experiments the time to reach equilibrium is several seconds, so this simple calculation suggests that the heat dissipated in this irreversible process can be an appreciable fraction of the condensation energy. This appears inconsistent with the experimental reports that indicate near perfect reversibility in these processes [5, 6, 8].

We can obtain a more accurate estimate of the energy dissipated using the procedure of Refs. [9, 14]. We assume the system is expelling a magnetic field $H_c(1-p)$ from its interior, with $p > 0$. As the phase boundary advances, the eddy currents induced raise the magnetic

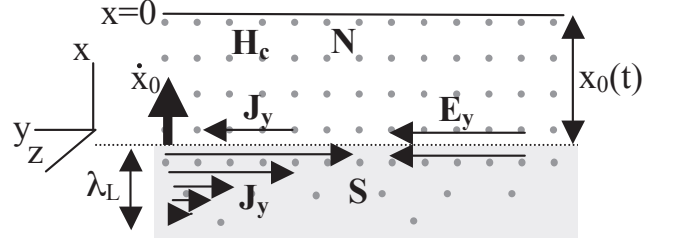


FIG. 3: The N-S phase boundary (dotted line) is moving up with speed \dot{x}_0 . The magnetic field (indicated by grey dots) points out of the paper. A Faraday field E_y is induced, which produces a current $J_y = \sigma E_y$ in the normal region flowing parallel to the phase boundary in the $+\hat{y}$ direction.

field to H_c at the phase boundary, and this limits the speed of growth of the superconducting phase. The time evolution of the phase boundary is given by [9]

$$x_0(t)^2 = R^2 - \frac{\alpha p c^2}{2\pi\sigma} t \quad (15)$$

where the parameter α is determined by the condition

$$\alpha \int_0^1 dy e^{\frac{\alpha p}{2}(y^2-1)} = 1. \quad (16)$$

The induced current in the normal region at position x and time t is given by [9]

$$J_y(x, t) = -\frac{c}{4\pi} \frac{\alpha p}{x_0} H_c e^{\frac{\alpha p}{2}((x/x_0(t))^2-1)} \quad (17)$$

and the total time for the phase boundary to move from $x_0 = -R$ to $x_0 = 0$ is given by

$$t_R = \frac{2\pi\sigma}{\alpha p c^2} R^2. \quad (18)$$

The energy dissipated per unit volume is

$$W = \frac{1}{R} \int_0^{t_R} dt \int_{x_0(t)}^0 dx J_y(x, t) E_y(x, t) \quad (19)$$

with $E_y = J_y/\sigma$, and a straightforward calculation yields

$$W = 2p \frac{H_c^2}{8\pi}. \quad (20)$$

Note that this result is very similar to what was obtained in the earlier calculation, since replacing p in Eq. (20) in terms of t_R (Eq. (18)) yields

$$W = \frac{H_c^2}{8\pi} \frac{4\pi\sigma}{\alpha c^2} \frac{R^2}{t_R} \quad (21)$$

the same as Eq. (10) except for the parameter α , which approaches 1 for $p \rightarrow 0$. For small p , $\alpha = 3/(3-p)$ [9].

Eq. (20) indicates that if the system is in the normal state at temperature $T < T_c$ in a magnetic field slightly

larger than $H_c(T)$ and the magnetic field is lowered for example to $0.95H_c$, in the process of becoming superconducting and expelling the magnetic field, an entire 10% of the condensation energy of the superconductor will be dissipated as heat. The same result Eq. (20) is obtained for the reverse process where the system is initially in the superconducting state in a magnetic field smaller than H_c and the magnetic field is increased to $H_c(1+p)$, causing the phase boundary to advance into the superconducting phase. The calculation assumes that the supercurrent is *not* dissipated in Joule heat when a region goes normal, however Joule heat is generated in the normal region because of the Faraday electric field resulting from the changing magnetic field. These results show that the transition from the superconducting to the normal state in the presence of a magnetic field, as well as the transition from the normal to the superconducting state in the presence of a magnetic field, cannot be reversible since the irreversible heat dissipated is given by Eq. (20) which is non-zero for any value of p , except $p = 0$ where no transition occurs. The irreversibility becomes smaller as the parameter p decreases and the time t_R over which the transition occurs Eq. (18) increases, but is always nonzero for any finite t_R .

The results discussed above were for a planar geometry where the calculations are simplest, however we have found that the results are very similar for a cylindrical geometry [9], and it is to be expected also in other geometries. Whether the system becomes superconducting through the process of expansion of a single domain as calculated here or through the (more realistic) process of creation of superconducting kernels in many locations at random that expand and merge, should not change the results. The essential fact is that to expel the magnetic field from the interior of a simply connected superconducting body, the magnetic field lines have to move through the entire body to the surface and the energy dissipation will only depend on the speed of the process and not on the details of domain growth.

V. THE PUZZLE

The derivation of the relation between change in entropy per unit volume and temperature derivative of critical field [4]

$$S_n - S_s = -\frac{1}{8\pi} \frac{\partial H_c^2}{\partial T} \quad (22)$$

rests on the assumption that the heat transfer to render the superconductor normal is given by

$$\delta Q = T dS \quad (23)$$

where dS is the change in entropy. In other words, that the process is reversible. The Rutgers relation for the specific heat jump at T_c

$$C_s(T_c) - C_n(T_c) = \frac{T_c}{4\pi} \left(\frac{\partial H_c}{\partial T} \right)_{T=T_c}^2 \quad (24)$$

follows from this equation. In addition, at any temperature $T < T_c$ the latent heat involved in the normal-superconductor transition for a sample of volume V is

$$Q = T(S_n - S_s)V = -VT \frac{H_c}{4\pi} \frac{\partial H_c}{\partial T} \quad (25)$$

assuming no irreversible increase in entropy takes place during the transition.

Keesom and coworkers did careful tests of the relation Eq. (25) for both the superconductor-normal [6] and the normal-superconductor [5] transitions, and found that it holds to great accuracy. As an example, they used an ellipsoidal sample of Sn ($T_c = 3.72K$) of dimensions 17.5 cm x 3.5 cm and measured the latent heat in the S-N transition at temperature 1.239K. Assuming for the conductivity of Sn at low temperatures $\sigma = 5 \times 10^8 \Omega^{-1} cm^{-1}$ [14] yields for Eq. (18)

$$t_R = \frac{\pi}{p} R(cm)^2 s \quad (26)$$

as the time it takes for the transition to take place in applied magnetic field $H_c(1+p)$ for sample dimension R . According to Eq. (20), this should result in an irreversible heat dissipation of $W = 2pH_c^2/8\pi$, or approximately

$$W \sim p \frac{T_c}{T} Q \quad (27)$$

with Q given by Eq. (25). According to Keesom and van Laer [6], Eq. (25) was satisfied to 0.1% accuracy in their experiment, which implies from Eq. (27) $pT_c/T < 0.001$, hence $p < 0.00033$. From Eq. (26) and assuming $R = 1.75cm$ (half the diameter of the ellipsoid) yields

$$t_R = 16,660s \quad (28)$$

or 4.6 hours for the duration of the experiment. Instead, according to Ref. [6], the experiment took only 696s. Conversely, we conclude that if the experiment took 696s the difference in the two sides of Eq. (25) should have been a factor of 24 larger than found by Keesom and van Laer. If the experiment was performed with a higher purity sample of Sn with up to two orders of magnitude smaller resistivity [15], the experiments would have to extend over 460 hours to show reversibility to 0.1% accuracy according to these estimates.

In summary, the experiments showed [5, 6] that the normal-superconductor transition is reversible to an accuracy much larger than expected for the duration of the experiments if eddy currents are generated in the normal region as the magnetic field changes as predicted by Faraday's law. In other words, the transition should be expected to exhibit far more irreversibility than was found in practice. How this can be explained is discussed in the next sections.

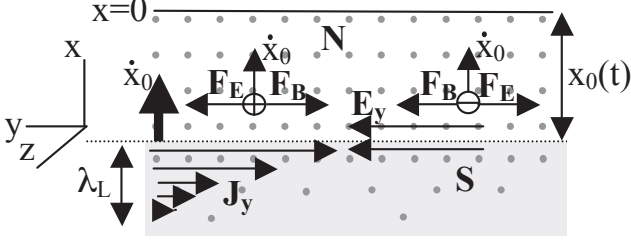


FIG. 4: The N-S phase boundary (dotted line) is moving up with speed \dot{x}_0 . The magnetic field (indicated by grey dots) points out of the paper. A Faraday field E_y is induced. If mobile charges (either electrons or holes) are drifting up at speed \dot{x}_0 they experience no net force in direction parallel to the phase boundary and hence no current J_y is generated.

VI. HOW TO RENDER THE NORMAL-SUPERCONDUCTOR TRANSITION REVERSIBLE

Consider again motion of the phase boundary into the normal region at speed \dot{x}_0 . We cannot negate Faraday's law, so an electric field $E_y = (H_c/c)\dot{x}_0$ will necessarily be generated at the boundary and by continuity in the normal region nearby. Is it possible that *no* current J_y is generated on the normal side of the phase boundary so as to avoid the irreversible Joule heating $J_y E_y$ resulting from it?

The answer is yes, provided the normal phase mobile charges are drifting at speed \dot{x}_0 in the same direction as the phase boundary. If so, the electric and magnetic forces are exactly balanced, both for positive and negative charges (holes or electrons) as shown schematically in Fig. 4, since for a charge q

$$\vec{F}_E \equiv qE_y\hat{y} = -\frac{q}{c}\dot{x}_0 H_c \hat{x} \times \hat{z} \equiv -\vec{F}_B \quad (29)$$

As a consequence, the phase boundary can move into the normal region as the superconducting phase expands *without* generation of Joule heat. The same argument holds for the opposite motion of the phase boundary into the superconducting region as the normal phase expands, with the normal charges now moving in opposite direction again following the motion of the phase boundary.

The flow of normal charges in direction perpendicular to the phase boundary depicted in Fig. 4 is precisely what is expected within the explanation of the Meissner effect provided by the theory of hole superconductivity [9, 10], as discussed in the next section.

VII. ORBIT EXPANSION AND BACKFLOW

We have argued in previous work that the perfect diamagnetism of superconductors implies that superfluid electrons reside in mesoscopic orbits of radius $2\lambda_L$ [11, 16]. The idea that superconducting carriers reside in large orbits was also proposed by several researchers

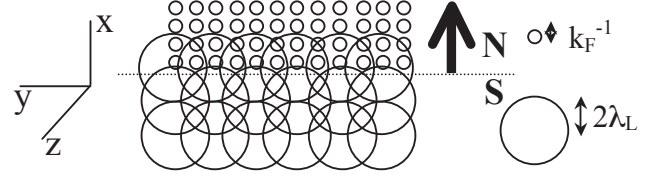


FIG. 5: Electronic orbits in the normal and superconducting states as proposed in the theory of hole superconductivity [12]. Note that large orbits centered in the superconducting side close to the phase boundary enter the normal side up to a distance $2\lambda_L$ from the phase boundary.

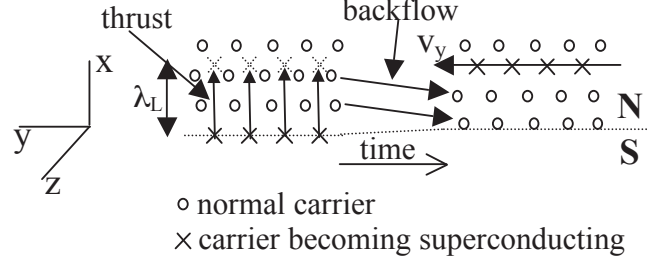


FIG. 6: As a consequence of the orbit enlargement when carriers become superconducting, negative charge thrusts from the superconducting into the normal region and in the process acquires through the Lorentz force a velocity \vec{v}_y in the $+\hat{y}$ direction. In addition, this generates a backflow of charge in the normal region.

in the pre-BCS era [17–19]. The Larmor diamagnetic susceptibility for electrons in orbits of radius k_F^{-1} and radius $2\lambda_L$ respectively yields the Landau diamagnetic susceptibility of normal metals and $(-1/4\pi)$, perfect diamagnetism, respectively [12], as appropriate for the normal and superconducting phases. This suggests that in the transition to the superconducting state, carriers expand their orbits from microscopic radius k_F^{-1} to radius $2\lambda_L$, and as the magnetic flux through the enlarging orbit increases the carrier acquires an azimuthal velocity generating a magnetic field opposite to the applied one. This provides a *dynamical* explanation for the origin of the Meissner current, that is not provided within conventional BCS theory [10]. In the planar geometry considered here, the orbits in the normal and superconducting state are shown schematically in Fig. 5. Note that the enlarged orbits in the superconducting region with centers at distance less than $2\lambda_L$ from the boundary enter partially into the normal region, up to a distance $2\lambda_L$ from the boundary into the normal region. Assuming the orbits correspond to electrons, this implies that negative charge enters from the superconducting into the normal region as an orbit next to the phase boundary expands.

The consequence of this orbit enlargement as carriers become superconducting for the process where the phase boundary moves into the normal region is shown schematically in Fig. 6 [9]. As new carriers (electrons) become superconducting and their orbits enlarge, nega-

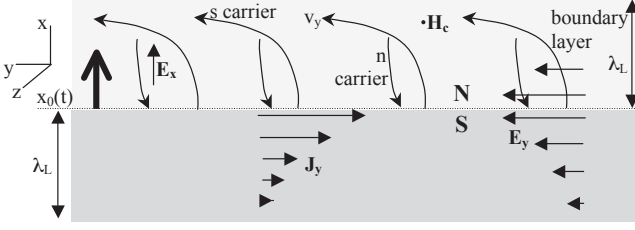


FIG. 7: As carriers become superconducting (s carriers) they thrust forward into the normal region over a boundary layer of thickness λ_L , and are deflected by the Lorentz force acquiring speed $v_y = -c/(4\pi n_s e \lambda_L) H_c$, in the $+\hat{y}$ direction assuming the s carriers are electrons. This process creates an electric field E_x in the $+\hat{x}$ direction that drives normal carrier (n carrier) backflow.

tive charge thrusts into the positive x direction and acquires through the Lorentz force momentum in the $+\hat{y}$ direction, resulting in speed v_y . The Lorentz force acting on an electron thrusting forward with speed v_x is

$$\vec{F}_L = -\frac{e}{c} v_x H_c \hat{y} \quad (30)$$

and the speed in the \hat{y} direction that an electron acquires in time Δt is

$$v_y = \int_0^{\Delta t} \frac{F_L}{m_e} dt = -\frac{e}{c} H_c \int_0^{\Delta t} v_x dt = -\frac{e}{c} H_c \Delta x \quad (31)$$

so that for $\Delta x = \lambda_L$

$$v_y = -\frac{e}{m_e c} \lambda_L H_c \quad (32)$$

which is precisely the speed of the carriers in the Meissner current Eq. (1). Under the assumption that $v_x \gg \dot{x}_0$, the effect of E_y on the forward thrusting electron can be ignored. This process then explains what drives the generation of the Meissner current flowing against the Faraday field E_y as the superconducting phase boundary advances into the normal phase. This physics also explains how in the reverse process, when the normal phase advances into the superconducting phase, the Meissner current stops without generating Joule heat, as will be discussed in detail in Sect. X.

This motion of negative (superconducting) charge into the normal region will create a charge imbalance and an electric field E_x will be generated in the normal region within distance λ_L from the phase boundary pointing in the $+\hat{x}$ direction, that will drive a flow of normal charge in the x direction, as shown schematically in Fig. 7. Fig. 7, reproduced from Ref. [10], assumes that the normal carriers (n carriers) are negatively charged electrons. This is incorrect, as discussed in the next section. In addition, the caption of this figure in Ref. [10] read “The normal (n) carriers do not acquire a large v_y in the opposite direction because they scatter off impurities and transfer their y -momentum to the lattice.” This is also incorrect, as we discuss in the subsequent section.

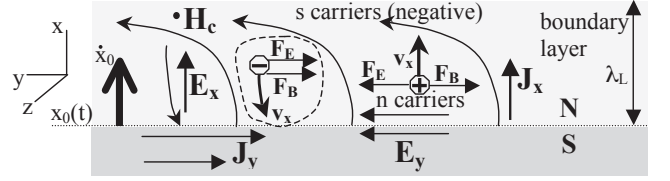


FIG. 8: Analysis of forces on the normal backflowing charge carriers (n carriers). If the carriers are electrons, electric and magnetic Lorentz forces F_E and F_B act in the same direction creating a current in the \hat{y} direction. We argue that this does not occur, hence have circled such a carrier with a dashed line. Instead, if the normal carriers are holes, electric and magnetic forces act in opposite directions and exactly cancel each other, generating no current in the \hat{y} direction. We argue that this is the situation in real materials.

VIII. THE SIGN OF THE NORMAL STATE CHARGE CARRIERS

Experiments that measure the gyromagnetic effect [20], the London moment [21] and the Bernoulli potential [22] in superconductors establish that the superconducting charge carriers are negatively charged. Therefore, as carriers at the boundary become superconducting *negative charge* is transferred into the normal region within a boundary layer, creating an electric field E_x pointing in the $+\hat{x}$ direction and a normal ‘backflow’ current J_x flowing in the $+\hat{x}$ direction. This backflow of normal carriers could occur through *negative electrons* moving in the $-\hat{x}$ direction or through *positive holes* moving in the $+\hat{x}$ direction.

Figure 8 shows the forces acting on the backflowing normal carriers. The speed of the normal carriers in the x direction, v_x , has to be \dot{x}_0 , the speed of motion of the phase boundary, so that no charge accumulation results. If the normal carriers are electrons, electric (F_E) and magnetic (F_B) Lorentz forces act in the same direction ($-\hat{y}$) as shown in Fig. 8, and this would create an eddy current in the $+\hat{y}$ direction generating entropy and rendering the process irreversible. Instead, if the normal state carriers (n carriers) are positive holes, electric and magnetic forces exactly cancel each other if the hole carriers are moving at the same speed $\dot{x}_0(t)$ as the phase boundary, as given by Eq. (29) and shown in Fig. 8.

This then implies that the normal carriers of materials that become superconducting in a reversible process and exhibit a Meissner effect are necessarily *holes*.

IX. RESOLUTION OF THE ANGULAR MOMENTUM PUZZLE

For several years we have been pointing out that the Meissner effect raises a puzzling question concerning angular momentum conservation [11, 12]. Consider a superconducting cylinder with axis in the \hat{z} direction to which a magnetic field in the $+\hat{z}$ direction is applied.

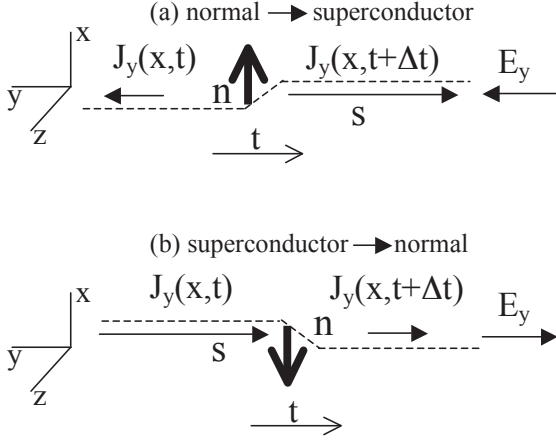


FIG. 11: (a) Phase boundary moves into the superconducting region, and (b) phase boundary moves into the normal region. The induced electric fields E_y point in opposite directions and so do the induced Foucault currents in the conventional description. In the superconducting region the current is the same for both cases. .

ties [10]. It generates no entropy and the process remains reversible.

X. EXPERIMENTAL TEST

Consider the growth of the superconducting phase in the presence of a magnetic field in a planar geometry as shown in Fig. 12. We have argued that a hole current J_x flows in a boundary layer of thickness λ_L in front of the moving phase boundary, as shown in Fig. 12 (a). Electric and magnetic forces on the normal hole carriers in the \hat{y} direction are balanced, as was shown in Fig. 8. This gives rise to a ‘Hall voltage’ V_H in the \hat{y} direction, that can be detected by placing contacts on the sample along a line parallel to the phase boundary that is approaching at speed \dot{x}_0 . If the distance between the contacts is d , the voltage measured will be $V_H = (H_c/c)\dot{x}_0 d$.

For example, for $H_c = 200G = 60,000V/cm$, the boundary moving at speed $\dot{x}_0 = 0.1cm/s$, and distance between contacts $d = 1cm$, the measured voltage will be $V_H = 0.6\mu V$. The voltage will appear during the time interval Δt where the boundary layer of thickness λ_L is moving across the x -position where the contacts are, $\Delta t = \lambda_L/\dot{x}_0$. For $\lambda_L = 500\text{\AA}$, $\Delta t = 50\mu s$. The polarity of the measured voltage will be as shown in Fig. 12, corresponding to a *positive* Hall voltage originating in conduction by holes. In contrast, if there is no charge flow in the \hat{x} direction as predicted by the conventional theory, the Faraday electric field E_y will cause charge to accumulate at the lateral edges of the sample, as shown in Fig. 12 (b), so as to exactly compensate the Faraday field E_y (positive (negative) charge to the left (right)), and the voltmeter will measure zero voltage throughout the process.

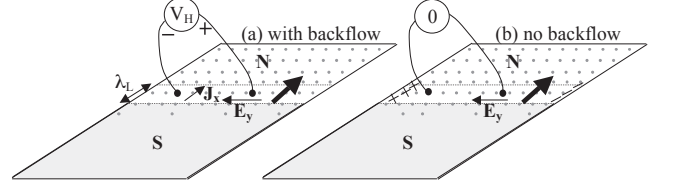


FIG. 12: In the presence of backflow, a voltmeter connected to the sample as shown in (a) will detect a Hall voltage V_H proportional to the speed of motion of the phase boundary, during the time interval where the boundary layer of thickness λ_L in front of the phase boundary moves across the region of the contacts. In the absence of backflow (b), no voltage will be measured.

For other geometries, similar differences in the expected results of such measurements predicted by our theory and the conventional theory may be expected.

XI. SUMMARY AND DISCUSSION

In previous work [9, 10] we have argued that to understand the dynamics of the Meissner effect it is necessary to assume that there is motion of charge in direction perpendicular to the normal-superconductor phase boundary, which is not expected within the conventional BCS-London theory of superconductivity. In this paper we have pointed out that such motion of charge also explains why the normal-superconductor transition is experimentally found to be reversible to high accuracy: the magnetic Lorentz force on normal charge carriers moving perpendicular to the phase boundary in a boundary layer cancels the tangential force due to the Faraday electric field that necessarily arises when the phase boundary moves, thus suppressing the generation of Joule heat by Foucault currents flowing parallel to the phase boundary. The physics discussed in this paper also explains how the Meissner current disappears in the process of the superconductor becoming normal without being dissipated as Joule heat.

It should be pointed out that some degree of irreversibility will always exist if there are regions of the sample that are not part of boundary layers, since in those regions eddy currents will flow. This will certainly be the case if for example a single domain grows from the center in a cylindrical sample as shown in Fig. 9. However if the growth of the superconducting phase occurs by formation of many separate domains that grow and subsequently merge, as shown schematically in Fig. 13, the fraction of the sample occupied by boundary layers correspondingly grows and Joule heating is reduced because no eddy currents flow within the boundary layers. Thus, one can envision a scenario where a sufficient number of domains start growing simultaneously so that any point in the sample is either in the superconducting state or in a boundary layer at any time, in which case entropy will not be generated and the transition will be com-

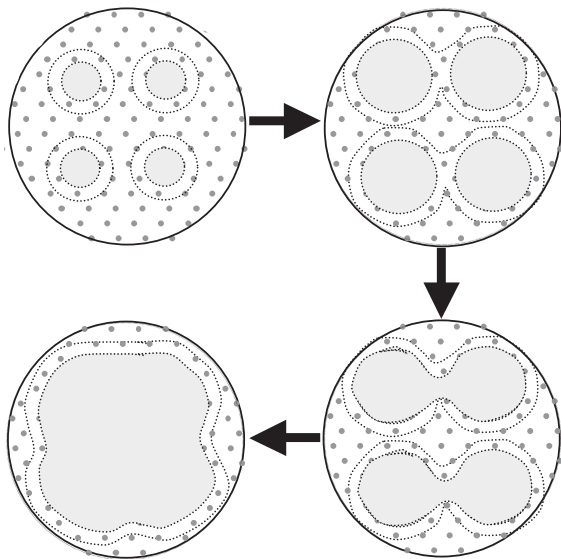


FIG. 13: Transition to the superconducting state (schematic) in the presence of a magnetic field (grey dots). Superconducting domains (grey areas) start at random places and grow outward, surrounded by a boundary layer (dotted lines) where current normal to the phase boundary exists and within which no tangential currents exist. The domains expand and merge until they occupy the entire sample.

pletely reversible and take a finite amount of time. This is *qualitatively* different from the conventional description where the transition *cannot* occur without entropy production.

Furthermore we showed that this suppression of eddy currents that would otherwise render the process irreversible will only occur if the carriers in the normal state are *holes*. Notably, this agrees with the prediction of the theory of hole superconductivity [23] that superconductivity can only occur if the normal state current carriers are holes.

Furthermore, we showed that this physics also explains the angular momentum puzzle pointed out in previous work [11, 12]. The flow of normal carriers in direction perpendicular to the phase boundary imparts momentum (or angular momentum) to the solid as a whole that exactly compensates the momentum (or angular momentum) gained by the carriers that become superconducting and carry the Meissner current, or lost by the superconducting carriers in the Meissner current that become normal. For this to happen it is also a necessary requirement that the carriers in the normal state are holes. This is because momentum transfer to the solid does not occur through scattering processes as we had earlier hypothesized [9, 12], but simply due to the fact that electrons of negative effective mass (holes) necessarily transfer momentum to the lattice when current flows [24, 25].

An essential aspect of the physics discussed here is that the wavefunction (‘orbit’) and the charge of superconducting electrons close to the normal-superconductor

phase boundary penetrates into the normal region. It is likely that this physics is related to the well known proximity effect, where signs of weak superconductivity are observed in junctions of superconducting and normal metals [27]. In the conventional explanations of that effect the superconducting order parameter is assumed to extend into the normal region [28] but this is not expected to be also associated with transfer of negative charge from the superconducting to the normal region, as in the physics described in this paper. In future work we will explore the applicability of the physics discussed here to describe proximity effect phenomena.

As discussed in this paper, the way in which a metal expels a magnetic field from its interior in the process of becoming superconducting, while conserving angular momentum and overcoming Faraday’s law, is far from trivial. We had pointed out in earlier work that expulsion of magnetic field necessarily has to involve a radial outward motion of charge [26]. However, as discussed in ref. [10], if it involved only outward motion of charge it would require the entire mobile charge in the metal to move out from the interior across the surface of the sample carrying the magnetic field lines with it, which obviously does not happen. Instead, the superconductor achieves this feat in a rather elaborate way, analogous to the mechanism of a ratchet wrench: the unrestricted motion corresponds to the outflow of electrons becoming superconducting over a distance λ_L , followed (although it happens concurrently) by the backflow of normal electrons that because of their antibonding character (negative effective mass) apply a force (or torque) to the entire body, and this combined flow and counterflow repeated over and over carries the magnetic field lines gradually out of the body over a macroscopic distance while conserving total momentum and angular momentum. In a sense it is as if all the mobile carriers flow out of the body as electrons and flow in again as antibonding electrons, or holes. It is not surprising that this process can take an extended period of time. It is also not surprising that the end result of this large amount of charge flowing out as light electrons and backflowing as heavy holes leaves as end result a small charge imbalance where some excess negative charge remains within the London penetration depth of the surface of the superconductor and some excess positive charge in the interior as predicted by the theory of hole superconductivity [29].

Instead, the conventional BCS-London theory of superconductivity explains the reversibility of the normal-superconductor transition in the presence of induced Faraday fields, the dynamics of the generation of the Meissner current in apparent violation of Faraday’s law, the transfer of momentum from the charge carriers to the lattice required to conserve momentum and angular momentum, and the disappearance of the Meissner current without irreversible heat loss, in a much simpler way: by simply postulating that it happens and that it needs no further explanation [30].

One way to test the physics discussed in this paper

would be to repeat the experiments measuring the latent heat in the superconducting transition in a field [5, 6] with very high purity samples [15], and verify that reversibility is satisfied to high accuracy for experiments extending over a period of time where the conventional

theory would predict a much higher degree of irreversibility, as discussed in Sect. V. Another way would be to detect Hall voltages as the phase boundary advances that would not be expected within the conventional theory, as discussed in Sect. X.

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